CAPACITIVE DISCONTINUITIES: RIGOROUS MULTIMODE EQUIVALENT NETWORK REPRESENTATION

M. Guglielmi
European Space Research and Technology Centre
Po. Box 299, Noordwijk, The Netherlands

Abstract

In this paper we present *novel, rigorous, multimode* equivalent network representations for a variety of zero-thickness capacitive windows and obstacles in a parallel plate waveguide. A key feature of these representations is that the coupling between all of the modes excited is described by a matrix whose elements *do not depend on frequency.* The value of the results presented is in that the networks developed can be used to *analyze rigorously* a large variety of *single and coupled planar transmission line structures* including radiation effects.

I. Introduction

The availability of accurate models for the design of planar, single or coupled, transmission lines is of great importance for the design of a large variety of microwave and millimeter-wave components and a number of techniques have been developed in the past for their analysis [1]. Of the various techniques that can be used, the transverse resonance technique is particularly attractive for its simplicity and versatility [8]. Following this technique, a planar transmission line in a metallic enclosure is seen as a transverse, capacitive discontinuity in a parallel-plate waveguide. For the full application of the transverse resonance concept, however, one must have at his disposal very accurate descriptions of the various discontinuities involved. Equivalent network representations are particularly valuable in this case because they fit naturally in to the transverse resonance approach.

Several equivalent network representations for capacitive discontinuities are available in the technical literature, for instance [2], [3] to cite a few. These network representations, however, are valid only under single mode conditions. Recently, we derived a set of multimode equivalent network representations for inductive discontinuities in rectangular waveguides [4]. In this paper, the procedure used in [4] is suitably modified and applied to the analysis of zero-thickness capacitive obstacles and windows.

The key component of the network representations derived is a multimode coupling matrix. The elements of this matrix are obtained in terms of the solution of a singular integral equation. The solution of this equation is shown to be known so that rigorous expressions for the matrix elements are derived. The analytic expressions obtained involve double integrations which are easily evaluated numerically.

A salient feature of this approach is that the coupling matrices derived do not depend on frequency. The frequency dependence of the whole equivalent network representation is only in terms of simple factors. The computations for the coupling matrices involve geometrical parameters and need to be computed only once for each given geometry. The analysis of the structures over a given frequency range involves only network computations and can be carried out very rapidly.

In addition to the key steps of the theoretical derivations, we also present numerical results. Computations are carried out to compare the results obtained by following our approach and those obtained by following already published data that are valid in the single mode case. Finally, it is discussed how the results obtained can be easily applied to the rigorous analysis of a very large class of planar transmission line structures.

II. Network formulation

The multimode network formulation outlined in this section has been applied to a variety of waveguide disconti-

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nuities (see Fig. 1). The general form of the multimode equivalent network representation obtained for all of the discontinuities treated is shown in Fig. 2. Closed form expressions for the coupling matrix elements for all the various cases have been derived for both TE and TM incidence and will be presented during the talk.

The formulation of the boundary value problem in terms of a multimode equivalent network representation follows similar steps for all cases. For the sake of space we will briefly outline here only the case of the capacitive strip shown in Fig. 1a. To begin, we recognize that since the discontinuity is uniform in the y direction, to solve the problem we can study the discontinuity with the excitation at normal incidence ($k_y = 0$ in Fig. 1). Once a network representation is derived, the result for the general case can be easily obtained by replacing k_o^2 with $k_o^2 - {k_y}^2$ everywhere [5]. The incident wave is chosen to be TM with respect to z. The first step is the expansion of the total transverse electric field in the form

$$\mathbf{E}_t^{(n)} = \sum_{m=0}^{\infty} V_m^{(n)}(z)\mathbf{e}_m(y) \qquad (1)$$

where $\mathbf{e}_m(y)$ are the orthonormal vector mode functions of the parallel plate waveguide [6] and the superscript (n) indicates $z \leq 0$ or $z \geq 0$ for $n = 1$ or $n = 2$, respectively. The next step is the imposition of the boundary conditions on the metal strip, obtaining

$$\sum_{m=0}^{\infty} V_m^{(1)}\mathbf{e}_m(x) = 0 \quad ; \quad z = 0 \qquad (2)$$

$$\sum_{m=0}^{\infty} V_m^{(2)}\mathbf{e}_m(x) = 0 \quad ; \quad z = 0 \qquad (3)$$

At this point one can add and subtract to the above series their behavior at infinity and, with a few manipulations, obtain the following expression

$$\sum_{m=0}^{\infty} \bar{V}_m\mathbf{e}_m(x) = \sum_{m=1}^{\infty} \bar{I}_m\frac{mB}{2}\mathbf{e}_m(x) \qquad (4)$$

where

$$B = \frac{-j\pi}{b\omega\epsilon_o} \qquad (5)$$

The presence of the constant B is due to the addition and subtraction of the behavior of the series at infinity, as discussed above. Furthermore, the modal voltage and currents $V_m^{(n)}$ and $I_m^{(n)}$ have been redefined according to

$$I_m^{(1)} - I_m^{(2)} = \bar{I}_m \qquad (6)$$

$$V_m + \bar{I}_m\frac{mB}{2} = \bar{V}_m \qquad (7)$$

In addition to modifying the boundary value equation, the step involving the behavior of the series at infinity has also an effect on the multimode equivalent network representation. In fact, in the case of the metal strip, one obtains the network in Fig. 3.

The next step in the solution of the problem is the use in (4) of the transform relation between the transverse magnetic field and the modal currents obtaining, after a few manipulations, the following integral equation

$$\mathbf{e}_m = \int_{d_1}^{d_2} M_m(y')\mathbf{A}_o\frac{B}{2}\sum_{n=1}^{\infty} n\mathbf{e}_n\mathbf{e}_n^* dy' \qquad (8)$$

where the unknown modal expansion functions M_m are related to the transverse magnetic field discontinuity due to the metal strip via

$$-\mathbf{z}_o \times (\mathbf{H}^{(1)} - \mathbf{H}^{(2)}) = \sum_{m=0}^{\infty} \bar{V}_m\mathbf{A}_o M_m(y') \qquad (9)$$

The completion of the network formulation is obtained by substituting the above expression into the transform relation between the magnetic field and the modal currents. The result of this last step is an expression for the generic element $Y_{n,m}$ of the multimode coupling matrix in Fig. 3, namely

$$Y_{n,m} = \int_{d_1}^{d_2} M_m(y')\mathbf{A}_o\mathbf{e}_n^* dy' \qquad (10)$$

It is important to note that the derivation of the fundamental integral equation has been carried without introducing any approximations. Furthermore, the kernel of (8) is not frequency dependent and this greatly simplifies its solution. The only frequency dependent term is B, as defined in (5). The resulting expression for the coupling matrix element has therefore the same, very simple frequency dependence.

The integral equation derived can be reduced to a Cauchy-type singular integral equation of known solution [7]. As a consequence, closed-form analytical expressions for the generic element of the multimode coupling matrix can be easily obtained. For the sake of space, the final expression is not reported here. It involves the double integration of simple trigonometric functions that can be easily carried out numerically. In the talk, the final form of the coupling matrix elements of all the discontinuities shown on Fig. 1 will be presented and discussed.

III. Applications

For a simple validation, the network obtained can be applied to the study of a capacitive strip in a rectangular waveguide. Figure 4 shows the results of a comparison between [3] and our multimode network approach in the single mode case. The quantity computed in the normalized susceptance seen by the lowest mode as a function of the obstacle width for a centered capacitive strip.

The main application of the results obtained is to the study of planar transmission lines. The networks developed can in fact be used to study planar transmission line geometries by using the transverse resonance technique [8]. Single and coupled lines can be easily studied. In addition, several discontinuities can be cascaded thereby expanding the range of applicability of this approach to a very large variety of structures. More details concerning this application will be presented during the talk.

IV. Conclusion

In this paper we present a set of *novel*, *multimode* equivalent network representations for a class of zero-thickness capacitive discontinuities. The main features of the results obtained are that they are *rigorous* and that the coupling matrix elements are *not frequency dependent* so that they only need to be computed once for each given geometry. The results presented can therefore be used to generate *numerically efficient* codes for the analysis of the frequency behavior of a very large variety of single or coupled planar transmission line structures.

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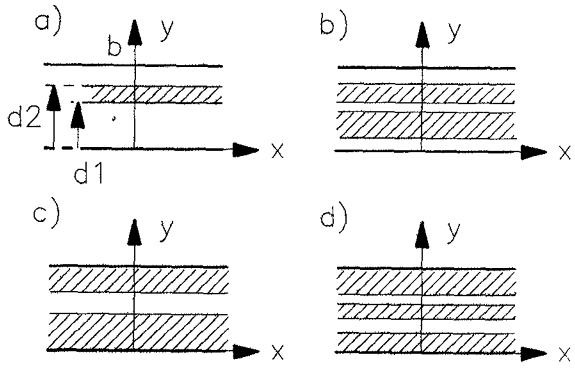

Fig. 1 Metallic, zero thickness capacitive discontinuities for which rigorous multimode equivalent network representations are derived.

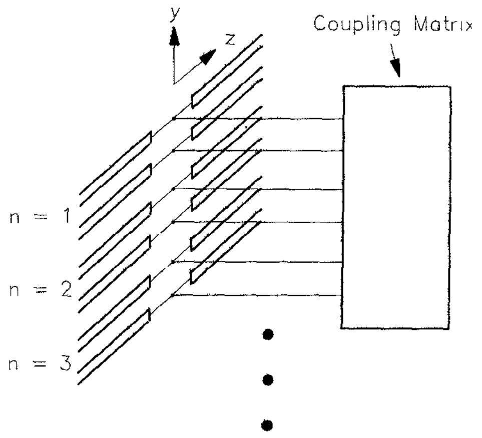

Fig. 2 General form of the multimode equivalent network representation for the discontinuities in Fig. 1.

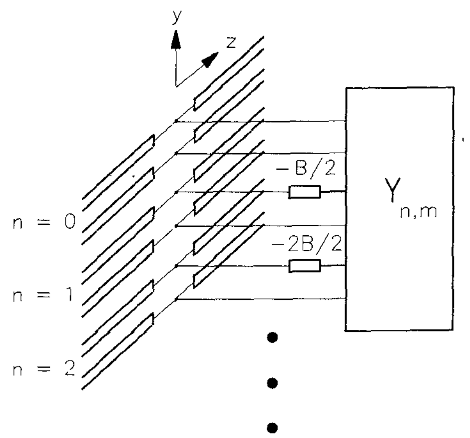

Fig. 3 Specific form of the equivalent network representation of the capacitive metallic obstacle in Fig. 1a.

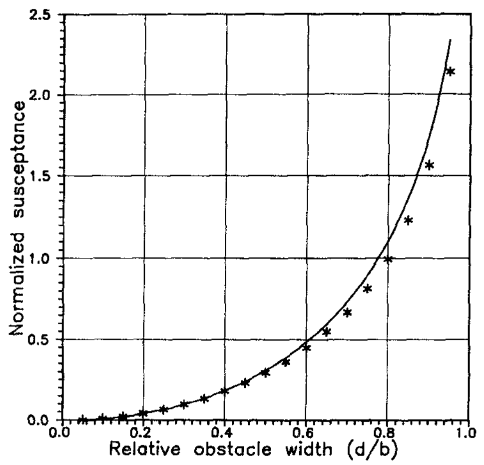

Fig. 4 Comparison of the normalized susceptance introduced by a centered capacitive strip in the lowest mode of a rectangular waveguide. The stars indicate the results obtained using [3] while the solid line has been obtained using the network presented in this paper. The waveguide dimensions are $a = 22.86mm$, $b = 10.16mm$ and the frequency is $9.0GHz$. The parameter d in this figure corresponds to $d2 - d1$ in Fig. 1a.